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PRODUCTION OF INOS IN THE BIG BANG

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ABSTRACT

The production of elementary particles in the big bang is discussed. In particular, the possibility that some elementary particle ("ino") is produced in the big bang and is present today in sufficient numbers to contribute to the overall mass density of the Universe in the form of dark matter is illustrated with several examples. The production of neutrinos, axions and baryons are covered in detail.

I. INTRODUCTION

The study of the structure of galaxies reveals the presence of a component of the total mass of the galaxy that is dark. Dark matter seems to be present not only in galactic halos, but also in the disk in the local vicinity of the solar system. Dark matter is also present in larger systems, such as binary galaxies, small groups of galaxies, clusters of galaxies, and perhaps in the Universe as a whole.[1.1]

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It is not clear if all the dark matter problems have the same solution. It may be that the dark matter in the disk is different than the dark matter in the halo, which in turn is different than the dark matter in clusters of galaxies, etc. It is also not clear whether baryons, either in the form of primordial black holes, jupiters, etc. could be some (or all) of the dark matter. Of particular cosmological interest is the possibility that some component of dark matter is non-baryonic, in the form of some elementary particle, or ino, which is a remnant of the big bang.

In this paper I will discuss the production of elementary particles in the big bang.[1,2] Some proposed candidates for ino dark matter are given in Table I. Possible masses range from 10^{-5}eV for axions to 10^{28}eV for pyrgons or Kaluza-Klein monopoles. The relic abundances of the particles if they are to contribute a significant fraction of the mass of the Universe are also given in Table I. One striking fact from the table is that there is a range of about 10^{33} in possible ino masses and abundances. Another striking fact is that particle physicists have been remarkably generous in providing candidate inos.

In this paper I will review the production of inos in the early Universe. After reviewing some general results from the standard model of the big bang, I will discuss the production of several of the candidates in Table I. I will first review the production of baryons, since it is obvious that baryons have something to do with galaxies. I will then discuss neutrinos, since other than baryons, they are the only ino in Table I that we know exists. I will then discuss axions. The neutrinos and axions are examples of "hot" and "cold" dark matter respectively.

TABLE I

SOME INO CANDIDATES FOR DARK MATTER

Candidate	Mass	Present Abundance
Axion	10^{-5}eV	10^{9}cm^{-3}
Neutrinos	10eV	10^{2}cm^{-3}
Gravitino/Photino	10^{3}eV	1 cm^{-3}
Baryons	10^{9}eV	10^{-6}cm^{-3}
Sneutrino/Photino	10^{11}eV	10^{-8}cm^{-3}
GUT Monopoles	10^{25}eV	10^{-22}cm^{-3}
Pyrgons/K.-K. Monopoles	10^{28}eV	10^{-25}cm^{-3}

II. THE BIG BANG MODEL

Although structure is observed in the Universe on very large scales, the structure seems to be superimposed on a smooth homogeneous background. Galaxies are not distributed randomly in the Universe, but they are correlated. The correlation may be quantified in the form of a two-point correlation function for galaxies, $\xi(r)$, which gives the excess probability of finding a galaxy a distance r from another.[2.1] If $\xi(r) \gg 1$, galaxies are strongly correlated on a scale r and are not distributed smoothly. If $|\xi(r)| < 1$, then galaxies can be well described as spread homogeneously throughout the Universe on the scale r. If $\xi(r) \ll -1$, galaxies are anti-correlated. The observations show that $\xi(r)$ decreases with increasing r and that $|\xi(r)| \leq 1$ on a scale of

$5h^{-1}$ Mpc,* i.e. on distance scales greater than $5h^{-1}$ Mpc the galaxy distribution is smooth to a good approximation.[2.1] If we assume that galaxies are a fair indication of mass, on scales greater than $5h^{-1}$ Mpc mass should be distributed in a homogeneous manner throughout the Universe.

The photons in the 3K microwave background[2.2] give us a sample of the Universe at large distances. Even if the photons are not truly primordial, the cosmic photosphere, or the surface of last scattering is certainly at cosmological distances. The mean free path for the microwave photons, λ, is related to the electron density, n_e, and the scattering cross section σ by

$$\lambda^{-1} = n_e \sigma \quad , \tag{2.1}$$

where the relevant cross section is the Tompson cross section, $\sigma = \sigma_T = 8\pi\alpha^2/3m_e^{\ 2} = 6.65 \times 10^{-25}\text{cm}^2$. The electron density is roughly half the baryon density, n_B. The baryon density is not well determined, but its value can be bracketed. It is convenient to express the baryon density in terms of a critical density, ρ_c

$$\rho_c = (3H_o^{\ 2})/(8\pi G) = 1.88 \times 10^{-29} h^2 \text{gcm}^{-3}, \tag{2.2}$$

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*The constant h reflects the uncertainty in the Hubble constant H_o, $H_o = 100h$ kms^{-1} Mpc^{-1}. It is expected that h is in the range $1 \leq h \leq 1/2$.

where H_o is the Hubble constant and G is Newton's constant. The baryon density, n_B is

$$n_B = 1.12 \times 10^{-5}\Omega_B h^2 cm^{-3} \quad , \tag{2.3}$$

where Ω_B is the ratio of the baryon density to the critical density,

$$\Omega_B \equiv \rho_B/\rho_c \quad . \tag{2.4}$$

The mean free path of the microwave photons is then

$$\lambda = \frac{1.3\times10^{29}}{\Omega_B h^2} \text{ cm} = \frac{4.2\times10^{4}}{\Omega_B h^2} \text{ Mpc} \quad . \tag{2.5}$$

Now this estimate for λ is a gross overestimate, since most electrons will be bound in neutral atoms. If we assume $\Omega_B h^2 \leq O(1)$, then the mean free path of the microwave photons is huge, and the photons must have had an origin at a very great distance in order to scatter and relax to a thermal distribution.

The microwave background is very nearly isotropic, i.e. the temperature is very nearly the same in all directions. On angular scales of about 4.5 arc minutes, a recent observation of Uson and Wilkinson[2.3] gives $\Delta T/T \leq 2.4 \times 10^{-5}$, where ΔT is a difference of the background temperature. On an angular scale of 180° there is a detected $\Delta T/T$ of about 10^{-3}, which could be the result of our galaxy having a peculiar velocity of 10^{-3}c. The observed isotropy of the microwave background suggests that out to cosmological distances the Universe is isotropic about us. If we believe that we do not live in a special place in the Universe, then the Universe should be isotropic about every point in the Universe. A space that is isotropic about every point is homogenous, so the microwave background implies that the Universe is homogeneous on large scales.

It should be stressed that a homogenous, isotropic Universe is not the only possibility.[2.4] There are many anisotropic cosmologies that can be constructed. In this paper I will only consider homogenous isotropic cosmologies. There are several advantages for considering only such cosmologies. The foremost reason as discussed above is that our Universe seems to be homogenous and isotropic. Another reason is that the symmetries of a homogenous, isotropic space allow a reduction of parameters in the metric. The fewer parameters in the theory, the better chance to interpret data. If the data can be understood by the simple homogeneous, isotropic model, then we have accomplished something truly remarkable, we have constructed a simple model for the large scale structure of the Universe. If the data cannot be understood by a homogeneous, isotropic model, then either the Copernican principle or General Relativity is incorrect, which would be an even more remarkable discovery.

If we assume the Universe is homogeneous and isotropic, it is possible to choose coordinates (r,θ,ϕ,t) for which the metric takes the form

$$ds^2 = dt^2 - R^2(t)\ \{dr^2/(1-kr^2) + r^2d\theta^2 + r^2\sin^2\theta d\phi^2\}\ , \tag{2.6}$$

where the cosmological scale factor $R(t)$ is a function of time. In the metric k is a constant, and it is possible to scale r such that $k = \pm 1, 0$. The spatial curvature scalar, ${}^3R$, is related to k and R by

$$ {}^3R = k/R^2(t) \quad . \tag{2.7} $$

If $k = 0$ the three space is flat, if $k = +1$ the three space has constant positive curvature, and if $k = -1$ the three space has constant negative curvature. The cosmological scale factor determines the proper distance between two fixed coordinates. The proper distance from the origin to coordinate r_1 is given by

$$ \begin{aligned} d_{PROP} &= R(t) \int_0^{r_1} dr \, (1-kr^2)^{-1/2} \\ &= R(t) \begin{cases} \sin^{-1} r_1 & k = \pm 1 \\ r_1 & k = 0 \\ \sinh^{-1} r_1 & k = -1 \quad . \end{cases} \end{aligned} \tag{2.8} $$

If $r_1 < 1$ (r_1 is dimensionless and scaled to R) then $d_{PROP} \simeq R(t) r_1$ for any k. The proper distance between any two comoving points scales with $R(t)$.

The generic behavior of $R(t)$ for $k = +1$, $k = -1$, or $k = 0$ is shown in Fig. 2.1. If $k = +1$ the Universe is closed, if $k = -1$ the Universe is open, if $k = 0$, the Universe is at the borderline.

Of course the metric is only half of the problem, the other half of the problem is the dreaded right hand side, $T_{\mu\nu}$. Again, we can use the symmetry of the problem to greatly restrict the form of $T_{\mu\nu}$. A particularly simple choice for $T_{\mu\nu}$ is the perfect form

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu - pg_{\mu\nu} \tag{2.9}$$

where ρ is the energy density, p is the pressure, and U_μ is the fluid velocity four vector. In the fluid rest frame $U_\mu = \delta^o_\mu$. The conservation of energy momentum $T^{\mu\nu}{}_{;\nu} = 0$ implies

$$(d/dr)(\rho R^3) = -3pR^2. \tag{2.10}$$

We will consider two simple forms for the equation of state; "matter" with p = 0, and "radiation" with p = $\rho/3$. Equation (2.10) then gives

$$\rho_R \propto R^{-4};\ \rho_M \propto R^{-3} \tag{2.11}$$

for the radiation and matter energy densities. For sufficiently small values of R the Universe is dominated by radiation. In discussing the production in inos in the big bang we will be in the radiation dominated era.

The radiation energy density receives a contribution not only from photons, but from all species of particles with mass smaller than the temperature. Therefore the radiation energy density is given by

$$\rho_R = (\pi^2/30)g_* T^4 \tag{2.12}$$

where g_* counts all species of particles with masses less than T, weighted by their spin degeneracy factors and a factor that depends on whether the particle is a boson or fermion

$$g_* = \sum_{\text{bosons}} g_B + (7/8) \sum_{\text{fermions}} g_F \quad . \tag{2.13}$$

With the choice of Eq. (2.9) for $T_{\mu\nu}$ and Eq. (2.6) for $g_{\mu\nu}$, the Einstein equations give

$$\begin{aligned} (\dot{R}/R)^2 + k/R^2 &= (8\pi G/3)\rho \\ &= \frac{4\pi^3}{45 m_{pl}^2} g_* T^4 \end{aligned} \tag{2.14}$$

where the last equality holds for $\rho_R > \rho_M$, and m_{pl} is the Planck mass ($G = m_{pl}^{-2}$). Since $T \sim R^{-4}$ for small R we can ignore the curvature term (kR^{-2}) and solve Eq. (2.14) for the time since infinite temperature as a function of temperature

$$t = (45/16\pi^3)^{1/2} g_*^{-1/2} m_{pl} T^{-2} \quad . \tag{2.15}$$

In discussing the decoupling of particles in the early Universe, reaction rates are compared to the expansion rate, H, given by

$$H = (\dot{R}/R) = (4\pi^3/45)^{1/2} g_*^{1/2} (T^2/m_{pl}) \quad . \tag{2.16}$$

The final ingredient in the standard model necessary to calculate ino production is conservation of entropy. The total entropy in a comoving volume is given by

$$S = s\,R^3 \tag{2.17}$$

where s is the entropy density given by

$$\begin{aligned} s &= (\rho + p)/T \\ &= (2\pi^2/45)\, g_* T^3 \quad . \end{aligned} \tag{2.18}$$

Note that if g_* changes as the temperature of the Universe falls below the mass of some particle, the temperature of the Universe will not scale exactly as R^{-1}, since $g_*(T)T^3R^3$ is constant.

III. PRODUCTION OF THE BARYON ASYMMETRY[3.1]

The overwhelming evidence is that if antimatter exists in the Universe in any appreciable amount, it must be separated from matter on a scale of clusters of galaxies, $10^{14} - 10^{15} M_\odot$.[3.2] Furthermore, this separation must be done in the early Universe when the temperature of the Universe was $T > M_N$, where M_N is the nucleon mass. If we would assume the Universe had zero net baryon number, $n_N = n_{\bar{N}}$, when $T \lesssim M_N$, the nucleons would annihilate with antinucleons.

The time evolution of the nucleon number density is given by

$$\frac{dn_N}{dt} = \frac{dn_{\bar{N}}}{dt} = -(n_N^2 - n_N^{eq\ 2})\sigma_A|v| - 3(\dot{R}/R)n_N \quad . \tag{3.1}$$

In Eq. (3.1) σ_A is the total nucleon-antinucleon annihilation cross section, and n_N^{eq} is the equilibrium abundance for a temperature T. The first term, $-n_N^2\sigma_A|v|$, accounts for the depletion of nucleon-antinucleon pairs by annihilation. The second term, $n_N^{eq\ 2}\sigma_A|v|$, accounts for $N\bar{N}$ creation in inverse annihilation processes. The final term, $-3(\dot{R}/R)n_N$, accounts for the dilution of the density due to the overall expansion of the Universe. Note that if $\sigma_A|v|$ is "large", the nucleons will track the equilibrium abundance, which becomes exponentially small for $T < M_N$. If $\sigma_A|v|$ is "small", the pair creation and annihilation terms will be unimportant and the nucleon abundance will change only due to the expansion of the Universe. Large and small $\sigma_A|v|$ refer to the relative size of the $n^2\sigma_A|v|$ term compared to the expansion term. When $n\sigma_A|v| < (\dot{R}/R)$, the reaction rates have "frozen out." If the reaction rates have frozen out, then the ratios of the nucleon density to entropy remains constant since $\dot{n}_N = -3(\dot{R}/R)n_N$ and $\dot{s} = -3(\dot{R}/R)s$, as can be seen from Eq. (2.17). Equation (3.1) can be used to calculate the development of the number density of any particle that can annihilate. The concept of freeze-out and conservation of number density relative to entropy is also quite general and will be used throughout this paper.

If Eq. (3.1) is solved for nucleons, the final baryon-to-entropy ratio is less than 10^{-18}. The entropy density today is related to the photon density by*

$$s = \frac{\pi^4}{45\zeta(3)} \{2 + (21/4)\ (T_\nu/T_\gamma)^3\} n_\gamma \tag{3.2}$$

$$\simeq 7n_\gamma$$

$$= 2800\ \mathrm{cm}^{-3} \quad . \qquad (T_\gamma = 2.7\mathrm{K}) \quad .$$

The nucleon density today is $n_N = 1.13 \times 10^{-5} \Omega_N h^2 \mathrm{cm}^{-3}$, and n_N/s is given by†

$$\frac{n_N}{s} = 4.0 \times 10^{-9} \Omega_N h^2 \quad . \tag{3.3}$$

Of course, the exact value of n_N/s is uncertain due to the appearance of $\Omega_N h^2$, but it is certainly much greater than 10^{-18}. Therefore if $n_N = n_{\bar{N}}$, annihilations would have reduced n_N far below its observed value unless N and $\bar{N}$ are separated before $T = M_N$.

Any attempt to separate N and $\bar{N}$ at early times must face the horizon problem. The distance over which causal processes can act is called the particle horizon. In the standard big-bang model the horizon distance is given by

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*In Eq. (3.2) we have assumed 3 families of 2-component neutrinos with temperature $T_\nu = 0.714T_\gamma$ (see the next section for a discussion of T_ν/T_γ).

†The baryon density is defined as $n_B = n_N - n_{\bar{N}}$. If $n_{\bar{N}} = 0$, then $n_B = n_N$. If $n_N = n_{\bar{N}}$, then $n_B = 0$.

$$d_H = 2t$$
$$= (45/4\pi^3)^{1/2} g_*^{-1/2} m_{pl} T^{-2} \quad . \tag{3.4}$$

At temperature $T = m_N = 10^3$MeV, the horizon distance is $1.4 \times 10^5 g_*^{-1/2}$cm. The entropy in the horizon volume is $S = s(4/3)\pi d_H^3 = (5/g_*\pi^3)^{1/2} \ (m_{pl}/T) = 7 \times 10^{57} g_*^{-1}$ at $T = m_N$. Even if the nucleon to entropy ratio was equal to one, there would be only $1M_\odot$ of baryons in the horizon at $T = M_N$, and non-causal processes would have to separate N and $\bar{N}$ on supercluster scales.

If, however, $n_B = n_N - n_{\bar{N}} \geq 0$ at $T \approx 1$GeV, then the Universe had an excess of nucleons over antinucleons and even perfect annihilation will leave a baryon excess. This excess is quantified as the baryon number B

$$B = \frac{n_B}{s} = \frac{n_N - n_{\bar{N}}}{s} \quad . \tag{3.5}$$

If we assume that today $n_{\bar{N}} << n_N$, then $B = 4 \times 10^{-9} \Omega_B h^2$ should be a conserved number as long as baryon number is conserved and the entropy is conserved. B of 10^{-9} is a rather curious number. At temperatures greater than M_N, there were roughly equal number of photons, baryons, and antibaryons, but for every ten billion antibaryons, there were ten billion and one baryons. The single extra baryon survives annihilation to become the baryons we observe in the Universe today.

One of the most remarkable advances in the field of particle physics and cosmology is the development of a model to account for the baryon asymmetry. The necessary ingredients for the generation of a baryon asymmetry from an initially symmetric state were first pointed out by Sakharov in 1967.[3.3] The three ingredients are 1) Baryon number violation, 2) C and CP violation, and 3) Non-equilibrium conditions.

The first condition, baryon number violation, is obvious. If baryon number is an exactly conserved quantum number in all interactions, $B \approx 10^{-10}$ must simply reflect the initial conditions. The second condition, C and CP violation is necessary since baryons are odd under C and CP. The third condition, non-equilibrium conditions is somewhat more subtle. If the baryon number can change, the chemical potential is not constant, and the entropy will be maximized when the baryon chemical potential (hence baryon number) is zero. Therefore if true chemical equilibrium is obtained, the baryon number will vanish.

Grand Unified Theories (GUTs), theories that unify the strong and electroweak interactions,[3.4] have baryon number violation. Although CP violation is not fully understood at low energies, it is supposed that CP and C violation will occur in GUTs. Finally the expansion of the Universe may be responsible for non-equilibrium. GUTs and the expansion of the early Universe offer a possible mechanism for the generation of the baryon asymmetry from an initially symmetric state. I will illustrate the mechanism by a simple model introduced by Kolb and Wolfram.[3.5]

Assume a model with a real massive boson X, and a massless species $b(\bar{b})$ with baryon number 1/2 (-(1/2)). In the absence of a Bose condensate or degenerate fermions we can use Maxwell-Boltzmann statistics. We will also assume that baryon-number conserving reactions will occur rapidly compared to the timescale for changing the baryon number. We may then write

$$f_b = \exp[-(E-\mu)/T]$$
$$f_{\bar{b}} = \exp[-(E+\mu)/T] \qquad (3.6)$$
$$f_X = \exp[-E/T]$$

where $E^2 = p^2 + m^2$ and the number density of a particle species is $n = \int f d^3p/(2\pi)^3$.

We model CP violation by writing the amplitudes for $X \leftrightarrow bb$, $X \leftrightarrow \bar{b}\bar{b}$ as (note: CPT requires $|M(i\to j)|^2 = |M(\bar{j}\to\bar{i})|^2$)

$$|M(X\to bb)|^2 = |M(\bar{b}\bar{b}\to X)|^2 = (1+\varepsilon)M_o^2/2$$
$$(3.7)$$
$$|M(X\to\bar{b}\bar{b}|^2 = |M(bb\to X)|^2 = (1-\varepsilon)M_o^2/2.$$

CP is violated in X decay if $\varepsilon \neq 0$.

The X number density evolves according to

$$\dot{n}_X = -\Gamma_X(n_X - n_X^{eq}) - 3(\dot{R}/R)n_X \qquad (3.8)$$

where n_X^{eq} is the equilibrium X abundance, and Γ_X is the total X-decay width ($\Gamma_X = |M_o|^2/16\pi$). The baryon number density, $n_B = n_b - n_{\bar{b}}$, evolves according to

$$\dot{n}_B = \varepsilon\Gamma_X(n_X - n_X^{eq}) - n_B n_X^{eq}\Gamma_X - n_B 2n(\sigma v) \qquad (3.9)$$

where $\langle\sigma v\rangle$ is the thermally averaged $2\leftrightarrow 2$ scattering cross section without real intermediate X states. Note that the driving term in $\dot{n}_B \propto \epsilon\Gamma_X(n_X - n_X^{eq})$ is non-vanishing only if CP is violated ($\epsilon \neq 0$), B is violated ($\Gamma_X \neq 0$), and non-equilibrium obtains ($n_X - n_X^{eq} \neq 0$).

The basic process for non-equilibrium is that the expansion of the Universe is too rapid for n_X to track its equilibrium value. This will occur when $T \approx m_X$ if the X interaction rate, $\propto \alpha m_X$, is less than the expansion rate, $\propto T^2 m_{pl}^{-1} = m_X^2 m_{pl}^{-1}$. This will happen only if

$$K = \alpha m_{pl}/m_X \tag{3.10}$$

is not too much larger than one. Analytic solutions give the final value of $B \propto K^{-1}$. If K is not too large m_X must be comparable to m_{pl}.

Detailed calculation of the baryon asymmetry in GUT models lead to a set of equations similar to Eqs. (3.8) and (3.9), albeit much more complicated.[3.5,3.6] The detailed studies show that an adequate baryon asymmetry can be generated for reasonable parameters (couplings, masses, CP violation) in the theories. It now seems likely that some sort of GUT interactions are responsible for generating the baryon asymmetry, hence generating the neutrons and protons that survive the early Universe to form the galaxies we observe.

IV. NEUTRINOS FROM THE BIG BANG

Neutrinos are neutral leptons, i.e. particles that only participate in weak interactions. In the early Universe neutrinos would have been produced in weak processes such as $e^+e^- \rightarrow \nu_i\bar{\nu}_i$ where the subscript i indicates the neutrino family, e, μ, or τ (or possibly more). If $E > m_e$, the cross section for neutrino production is

$$\sigma(e^+e^- \leftrightarrow \nu_i\bar{\nu}_i) \approx G_F^2E^2 \qquad (4.1)$$

where G_F is Fermi's constant. When $E > m_e$, the number density of electrons is given by $n_e \sim T^3$, so the production rate of neutrinos is $(E \approx T)$

$$\Gamma_p = n\sigma = T^3G_F^2E^2 \approx G_F^2T^5 \quad . \qquad (4.2)$$

This rate is to be compared with the expansion rate of the Universe $\Gamma_E \approx T^2/m_{pl}$

$$\begin{aligned}\frac{\Gamma_P}{\Gamma_E} &= G_F^2T^3m_{pl} \\ &= O(1)\ [T/1\text{MeV}]^3 \quad .\end{aligned} \qquad (4.3)$$

When the temperature of the Universe is greater than about 1 MeV, Γ_P/Γ_E is much greater than one and neutrinos interact; they are created and they are destroyed. The neutrinos would then be in equilibrium with the rest of the matter in the Universe. When the temperature of the Universe is less than about 1 MeV, Γ_P/Γ_E is much less than one and neutrinos "freeze out." After freeze-out they no longer interact and equilibrate with the rest of the Universe. A more detailed calculation can be done by solving Eq. (3.1) for neutrinos with $\sigma_A|v|$ the total neutrino annihilation cross section.[4.1]

If we assume that $m_\nu << 1$ MeV, the neutrinos will be relativistic at freeze out, and the number density of neutrinos (plus antineutrinos) at freeze out ($T=T_F$) would be

$$n_\nu = (\zeta(3)/\pi^2)\ (3/4)\ T_F^3 = (3/4)\ n_\gamma \tag{4.4}$$

where we have assumed 2-component neutrinos and n_γ is the number density of photons at freeze out.

The neutrinos decouple before e^+e^- annihilation. The e^+e^- annihilation increases the neutrino temperature by a factor of $(11/4)^{1/3}$ because the entropy in e^+e^- pairs is converted into photons but not neutrinos (since neutrinos have decoupled). Therefore the number density of neutrinos today, $n_{\nu o}$, is (per family)

$$n_{\nu o} = (3/4)\ (4/11)\ n_{\gamma o} = 110\text{cm}^{-3} \quad . \tag{4.5}$$

If the neutrino has a mass $m_{\nu i}$, then the relic neutrinos would contribute a fraction of the closure density

$$\Omega_{\nu i} = 0.01\ (m_{\nu i}/\text{eV})h^{-2} \quad . \tag{4.6}$$

If $h = 1/2$, $m_{\nu i}$ as low as 25eV could close the Universe. If we require $\Omega_{\nu i} \leq 1$, then $m_{\nu i} \leq 100h^2$eV. The limit $m_{\nu i} \leq 100$eV is much better than the present experimental bounds on $m_{\nu\mu}$ ($\lesssim$ 0.5MeV) and $m_{\nu\tau}$ ($\lesssim$ 164MeV).

The bound on m_ν has assumed that there are two degrees of freedom for ν in equilibrium at 1MeV, that there is a single species with a large mass, that the neutrinos are stable, and that m_ν < 1MeV. If there are more than two degrees of freedom for neutrinos and the other degrees of freedom interact with normal matter more weakly than usual, the bound on the mass has been studied by Olive and Turner. If the neutrino is unstable with a lifetime less than the age of the Universe, for a sufficiently short lifetime the massless decay products of the neutrino will give $\Omega \leq 1$. Finally if the neutrino is very massive its number density at freeze out will be exponentially supressed. In this case a neutrino with mass greater than about 2GeV will give $\Omega \leq 1$, even if stable.

From the particle physics point of view neutrinos are the most likely candidate ino to be important for galaxy formation. We know neutrinos exist! The standard Weinberg-Salam model has massless neutrinos, but there is no deep understanding (e.g. a symmetry principle) to explain why they should be massless. If neutrinos are stable ($\tau > t_u$) and have a mass in the 25-100eV range they will play an important role in the dynamics of galaxy formation.

V. AXIONS[5.1]

In the theory of strong interactions, QCD, it is possible to add to the usual Lagrangian

$$L_o = -(1/4)\, G^a_{\mu\nu}\, G^{a\mu\nu} \quad , \tag{5.1}$$

where $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon^{abc} A_\mu^{\ b} A_\nu^{\ c}$, a term of the form

$$L_\theta = (\theta/32\pi^2)\ \mathrm{tr}\ G^a_{\mu\nu}\,\tilde{G}^{a\mu\nu} \tag{5.2}$$

where $\tilde{G}^{a\mu\nu}$ is the dual of $G^a_{\mu\nu}$, $\tilde{G}^{a\mu\nu} = G^a_{\rho\sigma}\varepsilon^{\mu\nu\rho\sigma}$. It is possible to express L_θ as a total divergence, but unlike QED (where a similar term can be discarded as a surface term) it can have physical effects due to instantons. Since L_θ has the form ~ $\theta\vec{E}\cdot\vec{B}$, it violates P and T, hence it is odd under CP. One physical effect of the L_θ term would be a contribution to the neutron electric dipole moment. The fact that the neutron electric dipole moment is less than of order 10^{-19}e cm requires

$$(\theta/32\pi^2) \lesssim 10^{-8} \quad . \tag{5.3}$$

There is an additional contribution to θ. The quarks receive a mass when a Higgs field receives a vacuum expectation value $\langle\phi\rangle$. In general the coupling of ϕ to the quarks is neither real nor diagonal. When a rotation is performed to have the mass matrix real and diagonal, θ receives a contribution

$$\theta = \arg\det \underset{\sim}{M} \quad , \tag{5.4}$$

where $\underset{\sim}{M}$ is the quark mass matrix. Therefore the relevant parameter for CP violation is

$$\bar{\theta} = \theta + \arg\det \underset{\sim}{M} \quad . \tag{5.5}$$

The two terms in Eq. (5.5) have quite different origins, and it is necessary that they cancel to give $\bar{\theta} \lesssim 10^{-8}$. In order to understand this cancellation, Peccei and Quinn[5.2] introduced a global U(1) symmetry such that $\theta = -\arg\det \underset{\sim}{M}$ when $\phi = \langle\phi\rangle$. Thus θ is determined dynamically, and at the minimum of the Higgs potential, $\bar{\theta} \approx 0$. The $U(1)_{PQ}$ symmetry is broken by instanton effects. Weinberg[5.3] and Wilczek[5.4] pointed out that the spontaneous breaking of the $U(1)_{PQ}$ symmetry would lead to the appearance of a pseudo Nambu-Goldstone particle, called the axion. The axion is a pseudo Nambu-Goldstone particle since the $U(1)_{PQ}$ symmetry is not exact and is broken by instanton effects. Therefore the axion is not exactly massless, but picks up a mass

$$m_a = f_\pi m_\pi / v = 30\text{keV}\ (250\text{GeV}/v) \tag{5.6}$$

where the factor $f_\pi m_\pi$ comes from instanton effects, and v is the magnitude of the vacuum expectation of the Higgs field, $\langle\phi\rangle = ve^{i\alpha}$. In the original axion models ϕ was the Higgs responsible for the $SU_2 \times U_1$ weak breaking, but Kim[5.5] pointed out that it is not necessary to tie ϕ to the weak breaking. Throughout this section I will keep v arbitrary. Cosmological arguments will be able to bracket v to be in the range $10^8 - 10^{12}$GeV.

The axion couplings to fermions is

$$L_{ffa} \simeq (m_f/v)\ i\ \bar{f}\gamma_5 f a \tag{5.7}$$

where a is the axion field ($a = \text{Im}\ \phi$) and f is some fermion with mass m_f.

The axion also couples to photons through the anomaly

$$L_{\gamma\gamma a} \simeq (\alpha/3)v^{-1}\, F_{\mu\nu}\tilde{F}^{\mu\nu}a \tag{5.8}$$

where the F's are for the electromagnetic field. If $m_a < 2m_e$, the axion will decay to two photons with a lifetime

$$\tau(a \rightarrow \gamma\gamma) \simeq (v/f_\pi)^5\tau_\pi \tag{5.9}$$

where τ_π is the neutral pion lifetime. In order to hide the axion from detection, it is necessary to make v large. The properties of the invisible* axion are shown in Fig. (5.1).

The possible values of v can be limited by consideration of stellar evolution, in particular energy loss in red giant stars.[5,6] If axions can be produced in the core, they would escape the star causing an energy loss, and the nuclear fuel would have to be burned at a greater rate to compensate. If the loss is great enough the evolution of the red giant star would be too rapid to account for the observed numbers. Note that the mass of the axion is proportional to v^{-1}. If v is small enough, the axion would be too massive to be produced in the star. The axions are produced either through "Compton" emission, $\gamma+e\rightarrow a+e$, or through "Primakoff" emission, $\gamma+e\rightarrow\gamma+a$. In the first case the cross section for axion production depends upon the axion-electron coupling, while in the second case the cross section for axion production depends

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*Note that as $v \rightarrow \infty$, the axion decouples from the low energy theory. A model with a large v will decouple a, thus make it "invisible."

upon the axion-photon couplings. From Fig. (5.1) it is seen that both couplings are proportional to v^{-1}, so if v is large enough the axion production cross section will be small enough that axion emission is not a problem. If v is small enough the axion mass will be too large to be produced in red giants. Stellar evolution rules out $10^2\text{GeV} \lesssim v \lesssim 10^8\text{GeV}$.

Axions may play an important role in galaxy formation, since for certain values of v, Ω_a could be close to one, where Ω_a is the fraction of closure density in axions today.[5.7] When the temperature of the Universe was T > v, the finite temperature effects should have restored the symmetry,[5.8] and $\langle\phi\rangle=0$. When the temperature drops below $T \approx v$, a phase transition occurs and $|\langle\phi\rangle| = v$. However for temperatures $v \geq T \geq \Lambda_{QCD}$, where $\Lambda_{QCD} \approx 100\text{MeV}$, instanton effects are not important,[5.9] and the axion is a true Nambu-Goldstone particle. In this temperature regime the phase of $\langle\phi\rangle$ is irrelevant. When the temperature drops to $T \lesssim \Lambda_{QCD}$, the degenerate minima in $\langle\phi\rangle$ become noticeable and the axion field will evolve to one of the minima. The equation describing its evolution is (assuming the minimum at $\alpha=0$).

$$\ddot{\alpha} + 3H\dot{\alpha} + (\partial V/\partial\alpha) + \Gamma_a\dot{\alpha} = 0 \tag{5.10}$$

where $\partial V/\partial\alpha \approx m_a^2\alpha$. We can ignore the Γ_a term in (5.10) for the invisible axion. The axion mass in $\partial V/\partial\alpha$ is a function of temperature[5.9,5.7]

$$m_a(T) \approx (\Lambda^2/v)(\Lambda/T)^4\,[\ln(T/\Lambda)] \tag{5.11}$$

for temperatures $T \geq O(\Lambda)$.

The potential energy in the axion field due to the misalignment of α is

$$V(\alpha) = \alpha^2 m_a^2 v^2 = \alpha^2 \delta V. \tag{5.12}$$

If we assume m_a is a constant, then Eq. (5.10) tells us that

$$\alpha = \alpha_o A \cos(m_a t) \quad , \tag{5.13}$$

where $\alpha_o = \alpha(t_{QCD})$ and $A = (T/T_{QCD})^{3/2}$. This would correspond to an energy density today of

$$\rho_a = \alpha_o^2 10^{-22} g\ cm^{-3} \quad , \tag{5.14}$$

or about $10^7 \rho_c$ if $\alpha_o \approx 1$. However, m_a is not a constant and during the period that it changes the amplitude of the oscillation, A, is damped to keep the adiabatic invariant $A^2(t)m(t)$ constant. Even if $\alpha \approx 1$ at high temperatures α is damped to $\alpha_o^2 = 10^{-7} v_{12}{}^{7/6}$. Therefore, the true ρ_a today is[5.7]

$$\rho_a \approx 10^{-29} v_{12}^{7/6} g\ cm^{-3}. \tag{5.15}$$

From Eq. (5.15) we see that if $v_{12} > O(1)$, Ω_a would be greater than 1, and if $v_{12} \approx O(1)$, the Universe today would be dominated by a condensate of zero momentum axions.

VI. HOT AND COLD PARTICLES

Neutrinos and axions are examples of hot and cold dark matter respectively. The particle is hot or cold depending upon the velocity of the particle when the Universe becomes matter dominated.

Recall that the radiation energy density today is

$$\rho_{RO} = (\pi^2/30)g_* T^4 \qquad (T = 3K) \tag{6.1}$$

with g_* today given by

$$g_* = 2 + 2\cdot 3\cdot(7/8)\cdot(1/1.401)^4 \tag{6.2}$$

where the 2 is for photons, and the second term comes from three neutrinos with two degrees of freedom at a temperature $T_\nu = (1.401)^{-1}T_\gamma$. Since $\rho_R \sim R^{-4} = (1+z)^4$, the energy density in radiation at redshift z is

$$\rho_R = \rho_{RO}(1 + z)^4 \tag{6.3}$$

From Eq. (6.1) and Eq. (6.2)

$$\Omega_{RO} = 3.9 \times 10^{-5}h^{-2} \tag{6.4}$$

is the present fraction of the critical density in the form of radiation. If we assume there is some massive ino with $\Omega_M = \Omega_{MO}$ today, then

$$\rho_M = (1+z)^3 \Omega_{MO} \rho_c \tag{6.5}$$

and

$$\frac{\rho_M}{\rho_R} = \frac{2.6\times10^4 h^2 \Omega_{MO}}{1+z} \tag{6.6}$$

The crucial observation is that in deriving Eq. (6.6) we have not specified the identity of the dark matter, only that it gives a total Ω_{MO} today (we expect $\Omega_{MO} \simeq 0.9$ if the total $\Omega = 1$).

The importance of the ino velocity has to do with the damping of perturbations by free-streaming. Perturbations in the inos will suffer collisionless phase mixing on scales up to[6.1]

$$\ell_{DAMP} = H^{-1} v \tag{6.7}$$

where H^{-1} is the horizon and v is the particle velocity. Note that $H^{-1} \simeq t_u$, and that the damping scale increases until the particle becomes non-relativistic. The Universe becomes matter dominated at a temperature O(10eV), and if a particle has mass $m \lesssim O(3T) \approx 30$eV (for example neutrinos), structure up to H^{-1}(T=10eV) will be wiped out. This corresponds to a mass of about $3 \times 10^{15} M_\odot$.[6.1] If a particle is cold, $v \ll 1$, the damping scale will be much smaller.[6.2] The coldest particle is the axion, since it is a condensate of zero momentum particles.

The numerical simulations of the clustering[6.3] of relic inos depend upon the cosmological parameters (such as Ω, H_o, and the spectrum and

/ISIBLE AXION

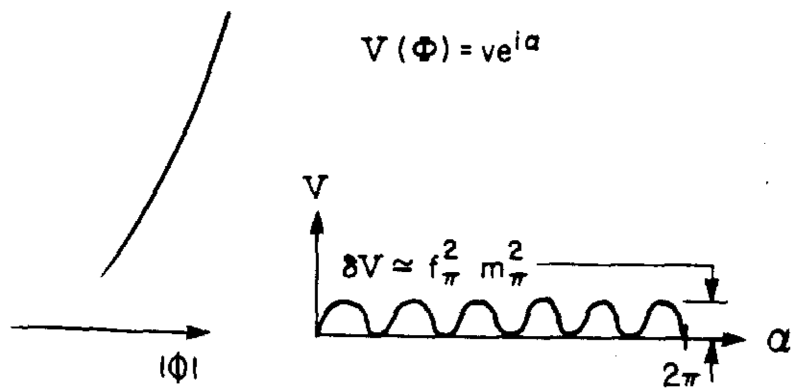
$\prime_{12} = v/10^{12}$ GeV)

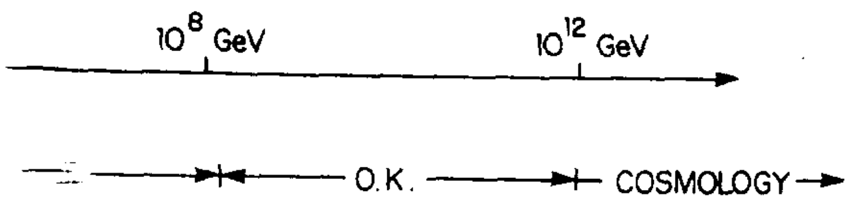
$$— \qquad \frac{10^{-15}}{v_{12}} \; \bar{e}\,\gamma_5\, e\, a$$

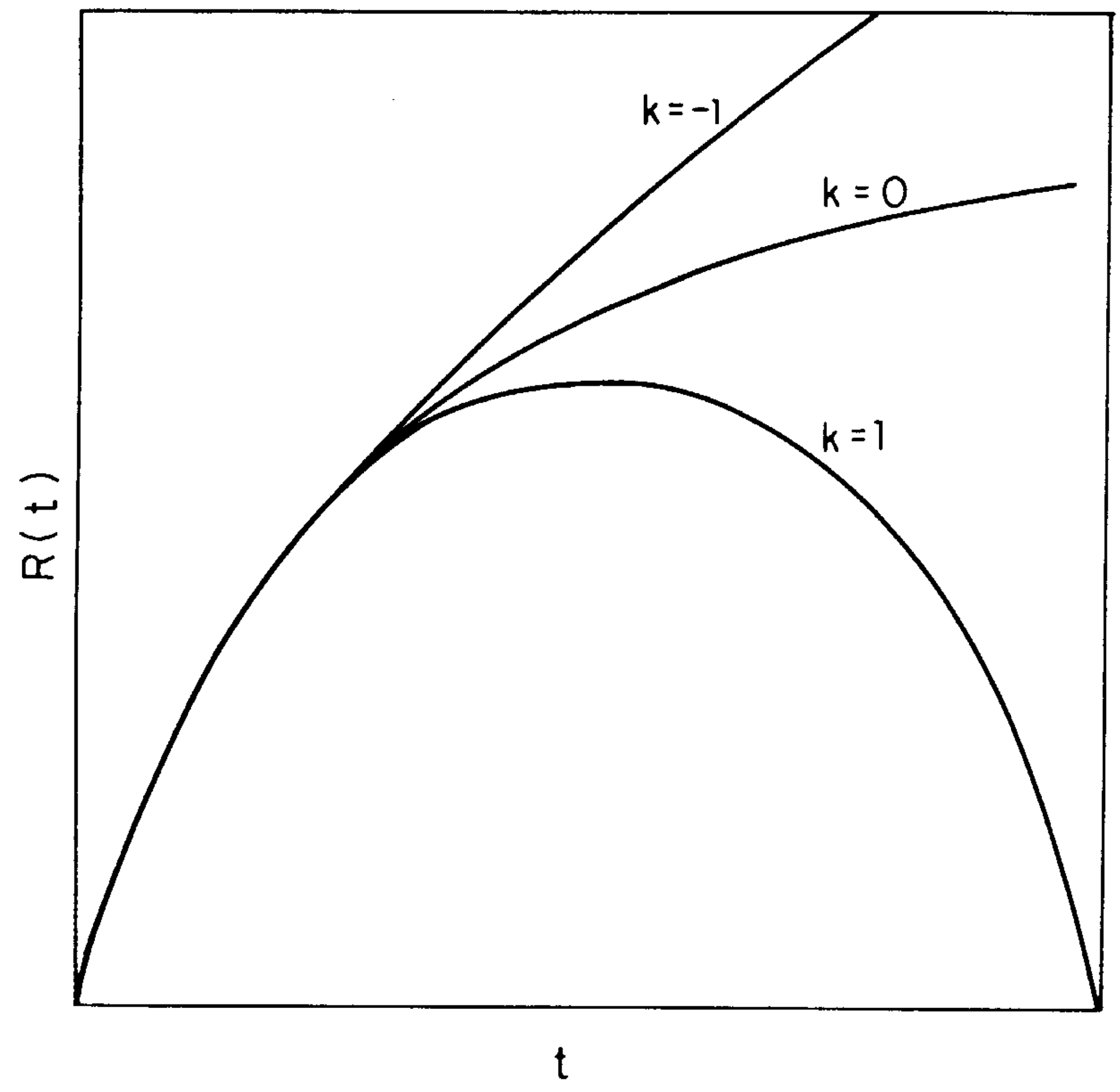
$$— \qquad \frac{2.4 \times 10^{-15}}{v_{12}} \; \mathrm{GeV}^{-1} \; \vec{E}_\gamma \cdot \vec{B}_\gamma$$

$$\frac{7.5 \times 10^{-16}}{v_{12}} \; \mathrm{eV}$$

$$10^{40} \; v_{12}^5 \; \text{years}$$

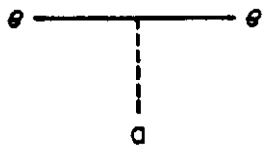




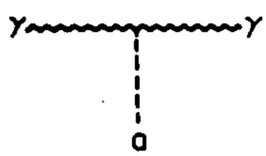

k = -1
k = 0
k = 1
R(t)
t

INVISIBLE AXION

$(v_{12} = v/10^{12}\ \text{GeV})$



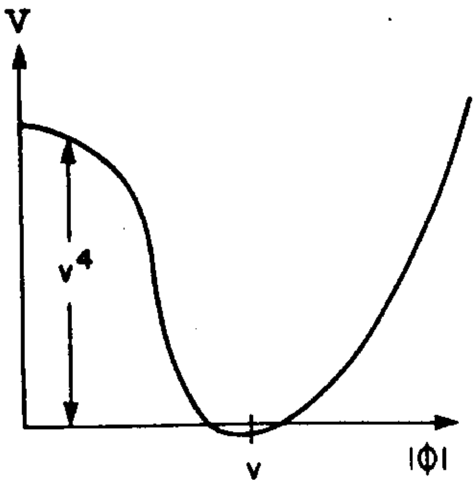
$$\frac{10^{-15}}{v_{12}}\ \bar{e}\gamma_5 e\, a$$



$$\frac{2.4\times 10^{-15}}{v_{12}}\ \text{GeV}^{-1}\ \vec{E}_\gamma\cdot\vec{B}_\gamma$$

m_a $$\frac{7.5\times 10^{-16}}{v_{12}}\ \text{eV}$$

$t\ (a\rightarrow 2\gamma)$ $$10^{40}\ v_{12}^5\ \text{years}$$



$V(\Phi) = ve^{ia}$